

Massive MIMO Receiver Design with Channel State Information

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Abstract: This paper investigates the sum-rate of the ongoing active research area of 5G Technology – Massive Multiple-Input Multiple-Output (MIMO) system receivers based on CSI. For the uplink, by assuming that the base station (BS) deploys zero-forcing (ZF) or maximum ratio combining (MRC) receivers, we present the achievable sum-rate for both receivers with imperfect and perfect channel state information (CSI). In case of Imperfect channel state information, to find the characteristics of channel, channel estimation is done by transmitting pilot symbols. Moreover, on the power scaling law, the impact of channel state information is also characterized and numerical analysis is obtained for combiner. The aim of this paper is to compare how receivers perform well in case of imperfect and perfect CSI and how increase in the number of antennas affects the power of the user.

Keywords: Massive MIMO, Channel State Information, Pilot Transmission, Power Scaling etc.

I. INTRODUCTION

Wireless communication technology has changed the way we communicate. The wired communication services like computers, Internet connections and telephones, should be used at preset locations, has gone. Nowadays these services are accessible wirelessly almost everywhere on Earth. The connectivity of devices through Wireless technology has necessary for the society like electricity. such technology itself produce new services and applications. The streaming media revolution witnessed by us, where music and video are delivered over the Internet. The first steps towards a fully networked society with connected homes, cars and machine-to-machine communications and augmented reality applications, have also been taken. And now a new communication technology is developed that is Massive MIMO (Multiple input and multiple output). 5G will need to be a paradigm shift that includes very high carrier frequencies with unprecedented numbers of antennas, extreme base station, massive bandwidths and device densities [1]. 5G Network targets an increase in data rates, device connectivity, coverage age and reduction in latency and energy consumption.

Channel State Information refers to the channel properties/characteristics in a communication link. This information describes signal propagation and the effect of scattering, fading and power decay over a distance makes it possible to adapt transmissions to current channel conditions, which are

crucial for reliable communication with high data rates. There are two types of CSI – *Perfect and Imperfect*. If the channel characteristics are known then it is Perfect, if not Imperfect. Channel characteristics refers to the fading, path loss, attenuation and Doppler spread.

2. UNDERSTANDING MIMO

MIMO (multiple input, multiple output) is one of the key technologies in 3G and 4G evolution. The antenna technology for wireless communications in which multiple antennas are used for wireless communications at both the source (transmitter) and the destination (receiver). The advantage of multiple antennas is increase in Diversity, which results in better reliability [3]. Data rates are improved by transmitting multiple data streams in parallel (Spatial Multiplexing). [4] As represented in figure below, MIMO has N number of transmitting and receiving antennas, all communicating with each other. Multiple paths here are represented by h . The MIMO system model can be represented as,

$$y = Hx + n$$

y is the received vector, x is the transmitted vector and n is noise vector.

Here,

$$\begin{bmatrix} y_1 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1t} \\ \vdots & \ddots & \vdots \\ h_{r1} & \cdots & h_{rt} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_t \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_r \end{bmatrix}$$

H is the channel matrix. The elements depend on the type of channel and its characteristics.

2.1 MIMO Receivers

We are considering two different MIMO receivers – Zero Forcing (ZF) receiver and Maximal Ratio Combiner (MRC). Zero Forcing receiver minimizes the effect of interference on the signal. It is modeled as,

$$\begin{aligned} \hat{x} &= \min ||y - Hx||^2 \\ \Rightarrow \hat{x} &= (H^H H)^{-1} H^H y \end{aligned}$$

Maximal Ratio Combiner (MRC) is used to maximize SNR. It is similar to a Matched Filter. Maximal Ratio Combiner is modeled based on receiver combining or beamforming. Beamforming is a fundamental technique in wireless communication used to focus a signal in a specific direction to improve its reliability and accuracy. [6]

It is given by,

$$w = \frac{h}{||h||}$$

3. Massive MIMO

Massive MIMO is a promising technology to enable high data rates in 5G. It provides an edge over 4G as the antennas are more in number. Massive MIMO pertains to addition of several hundreds to

thousands of antennas to achieve concentrated array gain. It is beneficial for increased throughput, reduced radiated power and greater simplicity in signal processing. Massive MIMO is further classified into Point-to-Point MIMO, Multi-User (MU) MIMO as discussed in [7]. In MU- MIMO, channel estimation becomes easier because the number of pilots required is proportional to number of users, which is smaller than the number of Base Station antennas. Massive MIMO operates in TDD (Time Division Duplexing) mode and, thus, channel estimated in the uplink can be used in downlink.[8] This is Channel Reciprocity and it impacts the performance of a Massive MIMO system [9].

3.1 Massive MIMO Uplink Processing

Consider an Uplink of MU-MIMO system here the cell has K single antenna users and base Station is equipped with M antennas. Let g_k denote the $M \times 1$ channel vector between base station and user k. Given by,

$$g_k = \begin{bmatrix} g_{1k} \\ g_{2k} \\ \vdots \\ g_{mk} \\ \vdots \\ g_{Mk} \end{bmatrix}$$

g_{mk} is the channel between m^{th} antenna of Base Station and k^{th} user

$$g_{mk} \sim C N(0, \beta_k)$$

$$E\{g_{mk}\} = 0, E\{|g_{mk}|^2\} = \beta_k$$

β_k models the attenuation and shadow fading, which is log normal of the random variable. The received vector y at the Base Station is given by where p_u is user power.

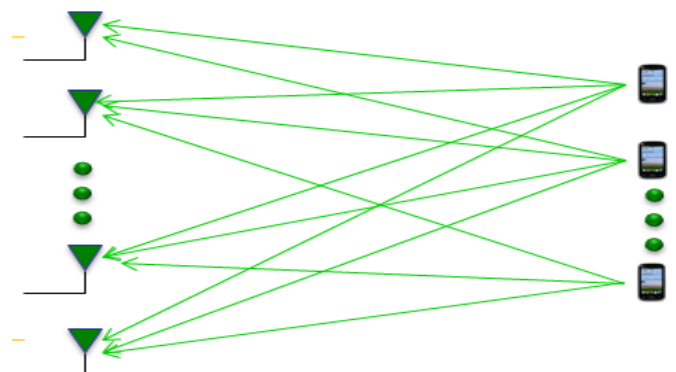


Fig -1: Massive MIMO Uplink Model

$$y = \sqrt{p_u} \underbrace{[g_1 \ g_2 \ \dots \ g_k]}_G \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}}_x + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_m \end{bmatrix}}_n$$

$$= \sqrt{p_u} Gx + n$$

Here, n is a vector of additive, zero-mean Gaussian noise. Noise samples are independent identically distributed. $n_m \sim CN(0,1)$

The concept of Channel Hardening as expressed in [10] fading decreases and becomes almost non-negligible. If g_{mk} is a complex Gaussian random variable of average power then we can model it as based on Weak Law of Large Numbers,

$$\frac{g_k^H g_k}{M} = \frac{\|g_k\|^2}{M} = \frac{|g_{1k}|^2 + |g_{2k}|^2 + \dots + |g_{Mk}|^2}{M} \rightarrow \beta_k$$

If you consider a beamforming vector, $w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix}$

and the quantity $w^H g_k$ is a complex Gaussian random variable $w^H g_k \sim CN(0, \beta_k \|w\|^2)$. Further, $\frac{w^H}{\|w\|} g_k \sim CN(0, \beta_k)$ equivalent to unit energy.

3.2 Receiver with CSI Certainty

Consider again $M \times 1$ received vector y at the Base Station

$$y = \sqrt{p_u} \underbrace{[g_1 \ g_2 \ \dots \ g_k]}_G \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}}_x + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_m \end{bmatrix}}_n$$

$$y = \sqrt{p_u} \sum_{i=1}^K g_i x_i + n$$

Taking user 1 as the desired user, the received signal is split into desired signal and interference.

$$y = \underbrace{\sqrt{p_u} g_1 x_1}_{\text{desired}} + \underbrace{\sqrt{p_u} \sum_{i=2}^K g_i x_i}_{\text{interference}} + n$$

The maximal ratio combiner receiver for user 1 as

$$\begin{aligned}
 r_1 &= \mathbf{w}^H \mathbf{y} \\
 r_1 &= \frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|} \mathbf{y} \\
 &= \frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|} \left(\sqrt{p_u} \mathbf{g}_1 x_1 + \sqrt{p_u} \sum_{i=2}^K \mathbf{g}_i x_i + n \right) \\
 &= \sqrt{p_u} \|\mathbf{g}_1\| x_1 + \sqrt{p_u} \sum_{i=2}^K \frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|} \mathbf{g}_i x_i + \frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|} n
 \end{aligned}$$

Now, SINR, which is the ratio of desired signal to interference and noise, is given as,

$$\text{SINR} = \frac{p_u \|\mathbf{g}_1\|^2}{p_u \sum_{i=2}^K E \left\{ \left| \frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|} \mathbf{g}_i \right|^2 \right\} + E \left\{ \left| \frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|} n \right|^2 \right\}}$$

As noise samples (n) are distributed as $CN(0,1)$ it follows

$$\frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|} n \sim CN(0,1) \Rightarrow E \left\{ \left| \frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|} n \right|^2 \right\} = 1$$

Coefficients of \mathbf{g}_i are distributed as $CN(0, \beta_i)$ it follows

$$\frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|} \mathbf{g}_i \sim CN(0, \beta_i) \Rightarrow E \left\{ \left| \frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|} \mathbf{g}_i \right|^2 \right\} = \beta_i$$

Therefore, SINR is obtained as

$$\text{SINR} = \frac{p_u \|\mathbf{g}_1\|^2}{p_u \sum_{i=2}^K \beta_i + 1}$$

3.3 Power Scaling for Perfect CSI

Consider now that power of each user is decreased inversely as number of Base Station antennas i.e., energy of user scaled with respect to M antennas.

$$p_u = \frac{E_u}{M}$$

SINR is further scaled as,

$$\begin{aligned} SINR &= \frac{p_u \|g_1\|^2}{p_u \sum_{i=2}^K \beta_i + 1} = \frac{\frac{E_u}{M} \|g_1\|^2}{\frac{E_u}{M} \sum_{i=2}^K \beta_i + 1} \\ &= \frac{E_u \frac{\|g_1\|^2}{M}}{E_u \left(\frac{1}{M} \sum_{i=2}^K \beta_i\right) + 1} \rightarrow E_u \beta_1 \end{aligned}$$

Hence, we can maintain constant SINR even with power decreasing as number of antennas increase. This is one of major advantages of Massive MIMO as it limits energy consumption. It also helps in suppression of effect of noise and Multi-user interference (MUI) on the desired signal.

$$\begin{aligned} SINR &= \frac{p_u \|g_1\|^2}{p_u \sum_{i=2}^K \beta_i + 1} \\ &\rightarrow \frac{p_u M \beta_1}{p_u \sum_{i=2}^K \beta_i + 1} = M \times \frac{p_u \beta_1}{p_u \sum_{i=2}^K \beta_i + 1} \end{aligned}$$

Based on channel hardening, $\|g_1\|^2 = M\beta_1$, by which we can observe that power of desired signal is improved by M times the MUI and noise power.

3.4 Pilot Transmission

The signal that is known to both transmitter and receiver used to know the channel properties is called pilot sequence.

The standard form of pilot matrix is:

$$\Phi = \begin{bmatrix} \varphi_1^H \\ \varphi_2^H \\ \varphi_3^H \\ \vdots \\ \varphi_K^H \end{bmatrix}$$

For easier computation of channel properties, the pilot matrix is chosen in such a way that:

$$\Phi \Phi^H = I$$

This is known orthogonal pilot matrix. The base station sends pilot signals through uplink to obtain the channel responses of user terminals. When received at the base station, the pilot signals are corrupted by noise and inter-cell interference. This process is called pilot contamination. There are three ways to reduce the Pilot contamination in massive MIMO systems: channel estimation, pre-coding, and pilot scheduling.

4. Channel estimation

As we know that in any wireless communications, the channel is the medium where the signal goes. While travelling through the channel the signal gets distorted or several noises added to the signal. At the receiver side the received signal is to be without these channel effects to get original information. In order to achieve this, we have to figure out the channel characteristics channel through which the signal travels. channel estimation is a process/technique to characterize the channel. There are many ways to do channel estimation as expressed in [11], but fundamental concepts are similar.

The model for channel estimation is:

$$Y_{M \times K} = \sqrt{p_p} G_{M \times K} \Phi_{K \times K} + N_{M \times K}$$

Here, K= number of pilot transmissions

$$\text{Pilot power}(p_p) = K p_u p_u = \text{user power}$$

Here, the pilot matrix is chosen in such a way that: $\Phi \Phi^H = I$

This is known as orthogonal pilot matrix. This is used to make the channel estimation easier.

Therefore, it follows:

$$\Phi \Phi^H = \begin{bmatrix} \varphi_1^H \\ \varphi_2^H \\ \varphi_3^H \\ \vdots \\ \varphi_K^H \end{bmatrix} [\varphi_1 \quad \varphi_2 \quad \varphi_3 \quad \dots \quad \varphi_K]$$

$$\begin{bmatrix} \|\varphi_1\|^2 & \dots & \varphi_1^H \varphi_K \\ \vdots & \ddots & \vdots \\ \varphi_K^H \varphi_1 & \dots & \|\varphi_K\|^2 \end{bmatrix} = I$$

Hence,

$$\|\varphi_i\|^2 = 1$$

$$\varphi_i^H \varphi_j = 0, i \neq j$$

Now, channel estimate can be obtained as:

$$\hat{G} = Y \frac{1}{\sqrt{p_p}} \Phi^H = (\sqrt{p_p} G \Phi + N) \frac{1}{\sqrt{p_p}} \Phi^H$$

$$\hat{G} = G + N \frac{1}{\sqrt{p_p}} \Phi^H = G + E$$

Here, channel estimation error is

$$E = \frac{1}{\sqrt{p_p}} N \Phi^H$$

We know that,

$$\|\varphi_i\|^2 = 1$$
$$n_1^H \varphi_2 \sim \text{CN}(0,1)$$

It follows that

$$n_1^H \varphi_2 \sim \text{CN}\left(0, \frac{1}{p_p}\right) = \text{CN}\left(0, \frac{1}{K p_u}\right)$$

Therefore, variance of channel estimation error is:

$$\frac{1}{K p_u}$$

Here, K is number of pilot symbol p_u is the power of user

4.1. Receiver with CSI Uncertainty

We know that massive MIMO model is

$$y = \sqrt{p_u} g_1 x_1 + \sqrt{p_u} \sum_{i=2}^K g_i x_i + n$$

The matched filter receiver for user 1 with CSI uncertainty is

$$r_1 = \hat{g}_1^H y$$
$$= \hat{g}_1^H \left(\sqrt{p_u} g_1 x_1 + \sqrt{p_u} \sum_{i=2}^K g_i x_i + n \right)$$

Hence, output with CSI uncertainty is

$$\begin{aligned} r_1 &= \sqrt{p_u} \hat{g}_1^H g_1 x_1 + \sqrt{p_u} \sum_{i=2}^K \hat{g}_1^H g_i x_i + \hat{g}_1^H n = \sqrt{p_u} (g_1 + e_1) g_1 x_1 + \sqrt{p_u} \sum_{i=2}^K \hat{g}_1^H g_i x_i + \hat{g}_1^H n \\ &= \sqrt{p_u} \|g_1\|^2 x_1 + \sqrt{p_u} e_1^H g_1 x_1 + \sqrt{p_u} \sum_{i=2}^K \hat{g}_1^H g_i x_i + \hat{g}_1^H n \end{aligned}$$

SINR is given by,

$$\text{SINR} = \frac{p_u \|g_1\|^2}{p_u \times \frac{1}{K p_u} + p_u \sum_{i=2}^K \frac{(\beta_1 + \frac{1}{K p_u}) \beta_i}{\beta_1} + \frac{(\beta_1 + \frac{1}{K p_u})}{\beta_1}}$$

4.2 Power Scaling for Imperfect CSI

Considering power scaling similar to Perfect CSI case,

$$p_u = \frac{E_u}{M}$$

The SINR is given as,

$$\text{SINR} = \frac{\frac{p_u}{M} \|g_1\|^2}{\frac{1}{K} + \frac{p_u}{M} \sum_{i=2}^K \frac{(\beta_1 + \frac{1}{K \frac{p_u}{M}}) \beta_i}{\beta_1} + \frac{(\beta_1 + \frac{1}{K \frac{p_u}{M}})}{\beta_1}}$$

Thus, the value of SINR is going to be

$$\begin{aligned} \text{SINR} &= \frac{E_u \|g_1\|^2}{\frac{1}{K} + \frac{E_u}{M} \sum_{i=2}^K \left(\beta_1 + \frac{M}{K E_u} \right) \frac{\beta_i}{\beta_1} + \frac{\left(\beta_1 + \frac{1}{K \frac{p_u}{M}} \right)}{\beta_1}} \\ &= \frac{E_u \beta_1}{\frac{1}{K} + \frac{E_u}{M} \sum_{i=2}^K \frac{1}{K} \frac{\beta_i}{\beta_1} + 1 + \frac{\left(\beta_1 + \frac{1}{K \frac{p_u}{M}} \right)}{\beta_1}} \rightarrow 0 \end{aligned}$$

Hence, we cannot scale power as

$$p_u = \frac{E_u}{M}$$

This is because, as transmit power decreases with increase in M in $p_u = \frac{E_u}{M}$, Channel estimation error increases as

$$\frac{1}{Kp_u} = \frac{M}{KE_u}$$

More error would affect the achievable rate as well as the accuracy. Now, considering power scaling as, where in M^α , $\alpha = 0.5$ to achieve a fixed non-zero achievable rate,

$$p_u = \frac{E_u}{\sqrt{M}}$$

The SINR is give as:

$$\begin{aligned} \text{SINR} &= \frac{\frac{p_u}{M} \|g_1\|^2}{\frac{1}{K} + \frac{p_u}{\sqrt{M}} \sum_{i=2}^K \frac{\left(\beta_1 + \frac{1}{K \frac{p_u}{\sqrt{M}}}\right) \beta_i}{\beta_1} + \frac{\left(\beta_1 + \frac{1}{K \frac{p_u}{\sqrt{M}}}\right)}{\beta_1}} \\ &= \frac{E_u \beta_1}{\frac{1}{K} + \sum_{i=2}^K \frac{1}{K} \frac{\beta_i}{\beta_1} + 1 + \frac{\left(\beta_1 + \frac{1}{K \frac{p_u}{\sqrt{M}}}\right)}{\beta_1}} \\ &\rightarrow K \beta_1 E_u^2 \frac{\|g_1\|^2}{M} \\ &\rightarrow K \beta_1 E_u^2 \end{aligned}$$

Thus, in order to keep the SINR as a non-zero value, the transmitted power can be scaled as

$$p_u = \frac{E_u}{\sqrt{M}}$$

Transmitted power only decreases as

$$p_u \propto \frac{1}{\sqrt{M}}$$

i.e. Power of the user decreases with increase in number of antennas as a factor of square root of number of antennas.

5. Results

For Perfect CSI, the theoretical achievable sum rate is give by, $R^{\text{mrc}} \rightarrow \log_2 (1 + E_u \beta_n)$, as $M \rightarrow \infty$ [Corollary 1, 12]. It is similar for R^{ZF} [Theorem 3, 12]. It shows that as the number of Base Station antennas grows, the uplink rate can be maintained while the transmit power can be substantially cut down. The achievable sum-rate converges to a non-zero limit when the number of antennas becomes large. But as observed, here ZF receiver is outperforming MRC receiver. MRC is a

simple signal processing since the Base Station just multiplies the received vector with the conjugate-transpose of the channel matrix H , and then detects each stream separately but MRC performs poorly in interference-limited cases as it cannot suppress intra-cell interference and neglects MUI. In zero-forcing receivers (ZF) do not perform well in noise limited cases. But it completely suppresses the effect of MUI as the received vector is multiplied by the pseudo-inverse of the channel matrix.

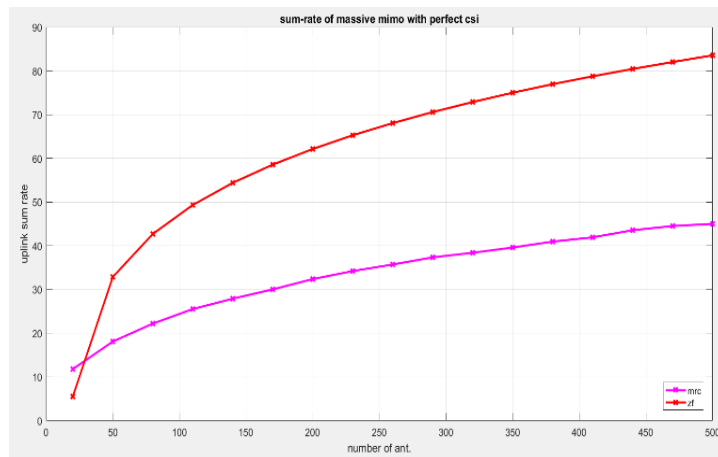


Fig-2: Uplink Sum-rate of Massive MIMO with Perfect CSI

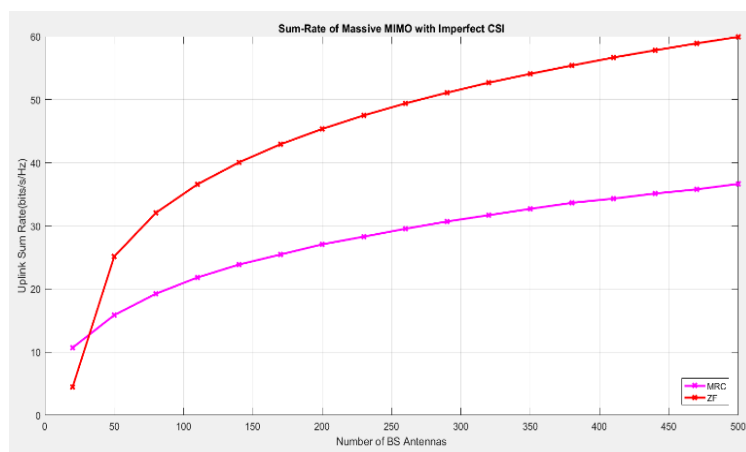


Fig-3: Uplink Sum-rate of Massive MIMO with Imperfect CSI

Similarly, for Imperfect CSI, ZF receiver is better than MRC receiver in terms of uplink sum-rate. But in Imperfect CSI, interference due to pilot contamination tends to dominate when shadowing is strong in MRC whereas in ZF it is zeroed out.

6. CONCLUSION

This paper has worked out the achievable uplink sum-rate of a Massive MIMO system in the case of both Perfect and Imperfect CSI. We have shown that, when the number of Base Station antennas M grow, we can decrease the transmitted power of user by $1/M$ if it is Perfect channel state information (CSI), and by $1/M^{1/2}$ if CSI is estimated using pilot symbols. Comparison between the two receivers in perfect and imperfect channel state information has been obtained and analyzed.

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